

# Josephson effect in the cuprates: microscopic implications

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## Abstract

In the tunnel limit, the current-phase relation of Josephson junctions can be expanded as  $I(\phi) = I_1 \sin \phi + I_2 \sin 2\phi$ . Standard BCS theory predicts that  $I_1 R_N \sim \Delta/e$  and  $I_2/I_1 \sim D$ , where  $R_N$  is the resistance of the junction in the normal state,  $\Delta$  is the superconducting gap, and  $D \ll 1$  is the junction transparency. In the cuprates, the experimental value of  $I_1 R_N$  ( $I_2/I_1$ ) is much smaller (larger) than the BCS prediction. We argue that both peculiarities of the cuprates can be explained by postulating quantum fluctuations of the pairing symmetry.

## 1 Introduction

The proximity to the metal-insulator transition is well known to lead to an anomalous normal state of the high  $T_c$  superconductors. It is therefore interesting to ask whether, apart from the  $d$ -wave symmetry of pairing, also the superconducting state of the cuprates can be regarded as unconventional. Most studies attempt to answer this question by considering the properties of *quasiparticle excitations*. For instance, photoemission experiments seem to support the conventional alternative, since sharp spectral functions have been observed at low temperatures [1]. On the other hand, the effect of strong correlations on the *condensate* has remained largely unexplored so far. This is surprising, since large quantum phase fluctuations are to be expected in the cuprates as a combined effect of the suppression of charge fluctuations and of the uncertainty principle. In particular, such fluctuations have been suggested to stabilize the RVB state and the presumably related pseudogap phase of the cuprates [2]. In this paper we argue that by a detailed analysis of the Josephson effect, new insights into the nature of quantum fluctuations in the cuprates can be obtained.

Josephson junctions involving the cuprate superconductors have been studied mainly because they enable phase-sensitive tests of pairing symmetry [3]. Attention has been paid especially to two particular types of Josephson junctions: grain boundary [4] and intrinsic [5] Josephson junctions. The best studied type of grain boundary Josephson junctions involves junctions

built on  $c$ -axis oriented films, where the weak link forms at the boundary of grains which are rotated around the  $[001]$  axis with respect to each other. Idealized junctions of this type are characterized by a planar interface and two angles  $\theta_1$  and  $\theta_2$  between the interface normal and the crystallographic directions in the grains forming the junction. The properties of the junctions depend dominantly on the misorientation angle  $\theta = \theta_2 - \theta_1$ . It is well known [4] that the transparency  $D$  of grain boundary junctions decays exponentially with increasing misorientation angle  $\theta$ ,  $D(\theta) \propto \exp(-\theta/\theta_0)$ . Thus, for  $\theta > \theta_0 \approx 5^\circ$ , grain boundary junctions are in the tunnel limit and (for  $\theta$  not too close to  $45^\circ$ , see Section 3) their current-phase relation  $I(\phi) = I_1 \sin \phi + I_2 \sin 2\phi + \dots$  can be well approximated by  $I(\phi) = I_1 \sin \phi$ , neglecting the higher-order harmonics.

## 2 The Josephson product

A useful quantity characterizing the superconducting electrodes forming the Josephson junction is the product of the first harmonic  $I_1$  with the junction resistance in the normal state,  $R_N$ . According to standard theory (for homogeneous featureless barriers), this so-called Josephson product is independent of the junction area and of the barrier transparency, thus giving an intrinsic information about the superconducting banks. It is well known that at interfaces between  $d$ -wave superconductors, anomalous bound Andreev levels may form [6]. At temperatures larger than the energy of such Andreev levels, BCS-like theory for rough interfaces between  $d$ -wave superconductors [7] predicts

$$(I_1 R_N)_{\text{BCS}} = (\pi/4)(\Delta/e) \cos 2\theta, \quad (1)$$

where  $\Delta$  is the maximal superconducting gap. The measured Josephson product of cuprate grain boundary junctions [4] can be well described by  $I_1 R_N = \alpha^2 (I_1 R_N)_{\text{BCS}}$  with a  $\theta$ -independent renormalization factor  $\alpha^2 \sim 10^{-1}$ . In addition to the  $\theta > \theta_0$  data of [4], this functional form can be tested also for  $\theta = 0$  (which case can be realized in break junctions) with the result that average Josephson products of such junctions [8] are fully consistent with the grain boundary data. Moreover, in [8] it has been shown that  $\Delta$  is not depressed in the junction region, thus explicitly demonstrating the breakdown of the BCS prediction for  $I_1 R_N$  in the cuprates.

There exists no generally accepted explanation of the small renormalization factor  $\alpha^2$ . One of the reasons is that the microstructure of Josephson junctions is typically quite complicated. In fact, it is well known that small

angle grain boundaries can be modelled by a sequence of edge dislocations, while at larger misorientation angles the dislocation cores start to overlap and no universal picture applies to the structure of the grain boundary. For large-angle grain boundaries, Halbritter has proposed [9] that the junction can be thought of as a nearly impenetrable barrier with randomly placed highly conductive channels across it. If due to strong Coulomb repulsion only the normal current (and no supercurrent) is supported by these channels, the small value of  $I_1 R_N$  follows quite naturally.

In this paper we shall argue that the smallness of the Josephson product does not follow from the particular properties of the barrier, but is rather an intrinsic property of the cuprates. Such a point of view has been first advocated in [10]. However, that paper did not consider alternative more conventional explanations. In order to support our point of view, let us begin by discussing the Josephson product for intrinsic Josephson junctions in the  $c$ -axis direction. Such junctions can be viewed as an analogue of  $ab$ -plane break junctions (since the misorientation angle vanishes for both), but are preferable because of simpler geometry of the interface. Moreover, zero energy surface bound states which may develop at  $ab$ -plane surfaces because of the  $d$ -wave symmetry of the pairing state [6] do not form in the  $c$ -axis direction, simplifying the analysis of intrinsic Josephson junctions.

Standard BCS theory applied to the case of tunneling between two-dimensional superconductors [11] predicts (for coherent  $c$ -axis tunneling) that the  $c$ -axis critical Josephson current density is  $j_1 = (2e/\hbar)N(0)\langle z_k t_k^2 \rangle$ , where  $\langle \dots \rangle$  denotes an average along the (two dimensional) Fermi line,  $z_k$  is the wavefunction renormalization, and  $t_k$  is the matrix element for  $c$ -axis tunneling between neighboring  $\text{CuO}_2$  planes.  $N(0)$  is the bare (unrenormalized) density of states,  $N(0) = \oint dk (4\pi^2 \hbar v_k)^{-1}$ , where the integration runs along the two dimensional Fermi line and  $v_k$  is the bare Fermi velocity at the Fermi surface point  $k$ . Note that although we have assumed that the self-energy is only frequency dependent, for this geometry  $z_k$  enters the expression for  $j_1$  (and also the normal-state resistance, see below).

Let us also note that the use of ordinary perturbation theory in deriving an expression for  $j_c$  has been criticized recently [12]. However, our formula yields the correct answer for  $t_k < \Delta$ . This can be shown either by an explicit solution of a  $4 \times 4$  Bogoliubov problem for two coupled planes with phase differences 0 and  $\pi$  between the planes, or by considering solutions to the gap equation in an infinite layered system with a finite  $c$ -axis total momentum of the Cooper pairs (for a similar calculation, see e.g. [13]).

The conductance per square in the normal state is given by  $G_N = (2e^2/\hbar)N(0)\langle z_k t_k^2 \Gamma_k^{-1} \rangle_N$ , where  $\Gamma_k$  is the inverse lifetime of the quasipar-

ticles and the index  $N$  in the Fermi line average means that the quantities are to be evaluated in the normal state. Therefore standard theory predicts for the Josephson product of intrinsic Josephson junctions  $I_1 R_N = j_1 G_N^{-1} \approx e^{-1} \langle z_k t_k^2 \rangle / \langle z_k t_k^2 \Gamma_k^{-1} \rangle_N$ . In conventional superconductors  $R_N$  can be measured at low temperatures in a sufficiently large magnetic field. This is impossible for the cuprates and thus  $R_N$  is usually defined as the  $c$ -axis resistivity at  $T_c$ .

Unfortunately, due to the unknown temperature dependence of  $z_k$ , the theoretical  $I_1 R_N$  can not be directly tested by experiment. In order to overcome this problem, in [14] instead of the usual Josephson product a related characteristic of intrinsic Josephson junctions has been studied, namely the product of the critical current  $I_1$  and of the resistivity  $R_S$  in the resistive mode of the junction (at low temperatures). Since the conductance per square in the resistive mode of the superconductor is [14]  $G_S \approx (8e^2/h)N(0)(z_k t_k^2)_{\text{node}}/\Delta$ , standard BCS-like theory predicts  $I_1 R_S = j_1 G_S^{-1} \approx (\pi/2)(\Delta/e)\langle z_k t_k^2 \rangle / (z_k t_k^2)_{\text{node}}$ . In [14],  $I_1 R_S \sim \Delta/e$  has been found experimentally and good agreement with theory has been claimed, since momentum-independent  $t_k$  and  $z_k$  were assumed. However, according to band structure calculations [15],  $t_k$  is strongly suppressed in the nodal directions. If this modulation of the tunnel matrix element  $t_k$  is taken into account and the presumably only moderate  $k$ -space dependence of  $z_k$  is neglected, the experimental  $I_1 R_S$  is seen to be drastically reduced with respect to the theoretical predictions.

Thus we have shown that although the barriers in grain boundaries and in intrinsic Josephson junctions are of very different nature, both types of junctions exhibit a suppressed Josephson product. Therefore we believe that this suppression is not due to specific barrier properties as suggested in [9], but rather due to some intrinsic property of the high- $T_c$  superconductors.

### 3 The second harmonic of the current-phase relation

Since the second harmonic  $I_2$  is not forced by symmetry to depend on the angles  $\theta_i$ , its Josephson product can be estimated (for temperatures larger than the energy of anomalous Andreev levels) using the standard BCS theory as  $I_2 R_N \sim D\Delta/e$ . Comparison with Eq. (1) implies that for junctions with  $\theta \approx 45^\circ$  the  $d$ -wave symmetry of pairing leads to a suppression of  $I_1$ , and  $I_2$  may become comparable to  $I_1$ . This has in fact been observed in two different types of  $45^\circ$  grain boundary Josephson junctions [16, 17].

However, the results of [16, 17] are quite mysterious, if we take into account the actual experimental setup. In fact, standard BCS theory with ideal featureless barriers implies that in order that  $|I_2| \sim |I_1|$ , the average misorientation angle  $\theta$  would have to be given with a precision  $\sim D$ , where  $D \sim 10^{-3}$ . This is not realistic and therefore two alternative explanations have been proposed, in both of which the origin of the anomalously large  $|I_2/I_1|$  ratio has been sought in the barrier properties.

(i) *Faceted scenario* has been considered as an alternative explanation for symmetric  $45^\circ$  junctions (i.e. junctions with nominal geometry  $\theta_1 = 0^\circ$  and  $\theta_2 = 45^\circ$ ), in which  $|I_2| > |I_1|$  has been found [16]. It takes into account the faceting of the grain boundary and also the twinned nature of the (orthorhombic) YBCO thin films. Due to both of these features, the junction can be viewed as a parallel set of 0 and  $\pi$  junctions [18]. It has been shown [19] that in such a case spontaneous currents are generated along the interface, the ground state energy of the junction is minimized at a macroscopic phase difference  $\pm\pi/2$ , and consequently the current-phase relation is dominated by the second harmonic  $I_2$ .

In what follows we analyze quantitatively whether the faceted scenario can apply to the results of [16]. Let us denote the current densities corresponding to the harmonics  $I_i$  (with  $i = 1, 2$ ) as  $j_i$  and introduce the Josephson penetration depth of the junction,  $\Lambda_J = (\Phi_0/4\pi\lambda\mu_0j_2)^{1/2}$ . Moreover, let the local critical current density in the 0 and  $\pi$  junctions be  $\pm j_0$ , their typical length  $a$ , and the bulk penetration depth be  $\lambda$ . Then, since  $a \approx 0.01 - 0.1\mu\text{m}$ ,  $\lambda \approx 0.15\mu\text{m}$ , and  $\Lambda_J \sim 3\mu\text{m}$  (estimated making use of [4]  $j_2 \sim 10^4 \text{ A/cm}^2$ ), the inequalities  $\pi\lambda \gg a$  and  $\Lambda_J^2 \gg a\lambda$  are well satisfied. Following [19] it is easy to show that these inequalities guarantee that the spontaneously generated currents along the Josephson junction can be calculated within perturbation theory. If we denote the total junction length by  $L$ , then a straightforward calculation yields  $j_2/j_0 \approx \mu_0j_0a\lambda^2/\Phi_0$  and  $j_1/j_0 \approx (a/L)^{1/2}$ . The equation for  $j_1$  is a random walk-type formula, indicating that  $j_1$  averages to zero in a sufficiently long junction. After some algebra the above equations are seen to imply  $j_2/j_1 \approx \sqrt{L\lambda/(4\pi\Lambda_J^2)}$ . Therefore, standard theory predicts that  $I_2 > I_1$  can be realized only in sufficiently long junctions with  $L > 4\pi\Lambda_J^2/\lambda$ . This requires  $L > 500\mu\text{m}$ , whereas in [16] much shorter junctions with  $L \sim 1\mu\text{m}$  were studied.

(ii) *Pinhole scenario* has been proposed in [17] as an explanation for symmetric  $45^\circ$  junctions (i.e. junctions nominally characterized by  $\theta_2 = -\theta_1 = 22.5^\circ$ ). It views the barrier as basically impenetrable, the conduction being due to randomly placed highly conductive pinholes. This explains

quite naturally the small value of the effective barrier transmission and, at the same time, the large value of  $|I_2/I_1|$ . Note that in order to explain the small value of the Josephson product, in addition to pinholes also Halbritter's conductive channels [9] have to be postulated, which are assumed to be highly conductive only in the normal and not in the superconducting channel. Because of this ad hoc nature of the pinhole scenario, and mainly because of the absence of higher harmonics in  $I(\phi)$  at 4 K [17] whose presence it predicts, we believe that the pinhole picture should be discarded.

Thus we conclude that the large value of the second harmonic  $I_2$  (compared with predictions of the standard BCS theory) is most probably not an extrinsic (barrier-related) effect, but rather an intrinsic property of the cuprates.

## 4 Microscopic implications

The two apparently unrelated experimental facts, namely the suppressed Josephson product  $I_1 R_N$  and the enhanced ratio  $|I_2/I_1|$ , can be explained by a single assumption that in the cuprates some mechanism is operative which leads to a suppression of  $I_1$ , while leaving  $R_N$  and  $I_2$  intact. In what follows we describe one such mechanism which we believe to be the most promising one. Namely, we suggest that at low temperatures the superconducting state of the cuprates supports fluctuations of pairing symmetry towards  $s$ -wave pairing (which pairing is expected to be locally stable within several microscopic models of the cuprates). Such fluctuations presumably do not affect  $R_N$ , while they do influence the Josephson current. In simplest terms, if we denote the phases of the superconducting grains forming the junction as  $\phi_i$ , then the fluctuations renormalize the first and second harmonics by the factors  $\langle e^{i\phi_1} \rangle \langle e^{i\phi_2} \rangle$  and  $\langle e^{2i\phi_1} \rangle \langle e^{2i\phi_2} \rangle$ , respectively, where  $\langle \dots \rangle$  denotes a ground-state expectation value. Thus experiment requires that the fluctuations have to be of such type that  $|\langle e^{i\phi} \rangle| = \alpha \approx 0.3$  and  $|\langle e^{2i\phi} \rangle| \approx 1$ . Precisely this behavior is expected if the  $d$ -wave order parameter fluctuates towards  $s$ -wave pairing.

Now we proceed by introducing a minimal model of such fluctuations. Unlike in standard literature on this subject (see, e.g., [20] and references therein), we assign a phase field  $\varphi_i$  to each bond  $i$  of the square Cu lattice. In other words,  $\varphi_i$  lives on the sites of the dual lattice. Since large phase fluctuations are expected, the compactness of phase fluctuations is explicitly

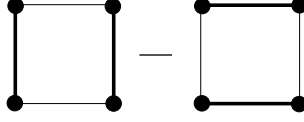


Figure 1: Ground state of the Heisenberg model on an elementary plaquette.

taken into account and the model reads

$$H = H_{\text{fluct}} + \sum_{\langle i,j \rangle} [-V \cos(2\varphi_i - 2\varphi_j) + W \cos(\varphi_i - \varphi_j)], \quad (2)$$

where  $H_{\text{fluct}} = -m^{-1} \sum_i \partial^2 / \partial \varphi_i^2$ . The second term in Eq. (2) (with  $\langle i, j \rangle$  denoting a pair of nearest neighbor sites) describes for  $0 < W < V$  a superconductor with a dominant  $d$ -wave pairing  $V + W$  and subdominant  $s$ -wave pairing  $V - W$ . Since we concentrate on the  $\mathbf{q} \approx (\pi, \pi)$  fluctuations from  $d$  to  $s$ -wave pairing, as a first approximation we do not take into account the coupling of phase fluctuations to electromagnetism.

In order to gain insight into the microscopics of  $H_{\text{fluct}}$ , let us recall that the RVB fluctuations in an elementary plaquette (in a hole-free region) favor a minus sign between the two different valence bond configurations (see Fig. 1). If these configurations are thought of in terms of Bose condensates of valence bonds, this means that energy is gained for a relative phase of the  $x$  and  $y$  condensates  $\arg(\sqrt{-1}) = \pm\pi/2$ . RVB processes are thus seen to frustrate the phase ordering dictated by the second term in Eq. (2).

Since the other singlet state of the elementary plaquette (with a plus sign between the valence bond configurations) lies higher in energy, the effective Hamiltonian in the singlet sector is  $H_{\text{RVB}} \propto \Delta_i^\dagger \Delta_{i+x+y}^\dagger \Delta_{i+x} \Delta_{i+y} + \text{H.C.}$ , where  $\Delta_i^\dagger$  creates a singlet on bond  $i$ . Neglecting the amplitude fluctuations of  $\Delta_i$ , we can write  $H_{\text{RVB}} = J \sum_i \cos(\varphi_i - \varphi_{i+x} + \varphi_{i+x+y} - \varphi_{i+y})$ . In what follows we replace  $H_{\text{fluct}}$  by  $H_{\text{RVB}}$  in the quantum model Eq. (2), thereby obtaining a completely classical toy model.

The toy model permits a simple mean field analysis: We consider two types of solution, both of which live on two sublattices A and B, and we explicitly disregard four-sublattice solutions, since they correspond to translation-symmetry breaking states on the original Cu lattice. In the first type of solution  $\varphi_A = \varphi$  and  $\varphi_B = 0$  and there are two possibilities: either  $\varphi = \pi$  ( $d$ -wave), or  $\varphi < \pi$  (a complex mixture of  $d$  and  $s$ , to be called  $d+is$ ). In the second type of solution we assume a disordered state of such type that on sublattice A, the phase equals 0 and  $\pi$  with probabilities  $(1 + \alpha)/2$  and

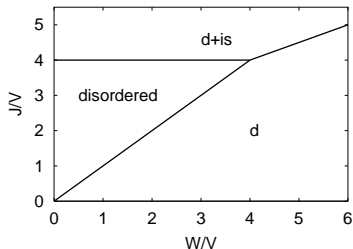


Figure 2: Mean field phase diagram of the toy model.

$(1 - \alpha)/2$ , respectively, and on sublattice  $B$  the values 0 and  $\pi$  are interchanged. In this case the macroscopic symmetry is of the  $d$ -wave type, with renormalized averages  $\langle e^{i\varphi_A} \rangle = -\langle e^{i\varphi_B} \rangle = \alpha$  and  $\langle e^{2i\varphi_A} \rangle = \langle e^{2i\varphi_B} \rangle = 1$ . Minimization of energy (with respect to  $\varphi$  or  $\alpha$ ) leads to the phase diagram shown in Fig. 2. The renormalization factor in the disordered  $d$ -wave state (which presumably corresponds to a homogeneous but strongly fluctuating  $d$ -wave phase in the model Eq. (2)) is  $\alpha = (W/J)^{1/2}$ . Thus the cuprates correspond to the region  $W \approx J/10$  and  $J < 4V$  in Fig. 2.

## 5 Conclusions

The anomalous effects observed in cuprate grain boundary and intrinsic Josephson junctions can be explained by a single assumption of strong quantum phase fluctuations at  $\mathbf{q} = (\pi, \pi)$ , which leads to  $|\langle e^{i\phi} \rangle| = \alpha \approx 0.3$  and  $|\langle e^{2i\phi} \rangle| \approx 1$ . In addition, our picture implies that the Josephson product for junctions between the cuprates and low- $T_c$  superconductors is renormalized by the factor  $\alpha$ , in semiquantitative agreement with experiment [21]. It also may be relevant for the experiment [22], where a large second harmonic has been found in a  $c$ -axis Josephson junction between YBCO and Nb.

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